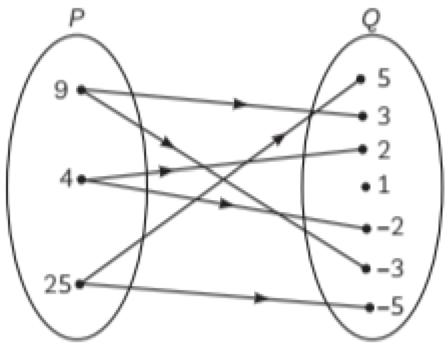
Relations and Functions

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. A class XI teacher, after teaching the topic of 'Relations; tries to assess the performance of her students over this topic. The figure shows a relation between the sets P and Q.



(A) This relation in set builder form is:

(a) R = {(x, y): x is square root of y, x ∈P, y ∈Q}

(b) R= {(x, y): y is square of x, x ∈P, y ∈Q}

(c) R= {(x,y): x is square of y, x ∈P, y ∈Q}

(d) none of these

(B) The domain of relation is:

(a) (1, 2, 3, 4, 5}

(b) {4, 9, 25, 5}

(c) (4,9}

(d) {4, 9, 25}

(C) The range of relation is:

(a) {4, 9, 25}

(b) {1, 2, 3, 4, 5}

(c) (-2, 2, -3, 3, -5, 5}

(d) -5, -3, -2, 1, 2, 3, 5}

(D) This relation in roster form is:

(a) (9, 3), (4, 2), (25, 5)}

(b) (9, 3), (9, -3), (4, 2), (4, -2), (25, 5), (25,-5)}

(c) {(9,-3), (4, -2), (25,-5)}
(d) none of the above

(E) The total number of relation from set P are:

(a) 32
(b) 64
(c) 128
(d) none of these

Ans. (A) (c) R= {(x,y): x is square of y, x∈P, y∈Q}
Explanation: Relation R is "x is the square of y".
.. In set builder form, R = {(x, y): x is the square of y, x∈P, y∈Q}.
(B) (d) (4, 9, 25}
Explanation: The domain of relation is an element of set Pi.e. {4, 9, 25}.
(C) (c) (-2, 2, -3, 3, -5, 5}
Explanation: The range of relation is {-2, 2,-3, 3, -5, 5}.
(D) (b) (9, 3), (9, -3), (4, 2), (4, -2), (25, 5),(25,-5)}
Explanation: In roster form R = {(9, 3).
(9,-3), (4, 2), (4,-2), (25, 5), (25,-5)}.
(E) (b) 64
Explanation: Total number of ordered pair in R= 6 (note that total no. of ordered pairs possible are 3 x7 = 21)
.. Total number of relation = 26 = 64

2. Method to find the sets when cartesian product is given.
For finding these two sets, we write the first element of each ordered pair in first set say A and corresponding second element in second set B (say). Number of elements in cartesian product of two sets.
If there are p elements in set A and q elements in set B, then there will be pq elements in A x B i.e., if n (A) = p and n (B) = q, then n (A x B) = pq.
(A) If A x B = {(a, 1), (b, 3), (a,3), (b, 1), (a, 2), (b,2)}. Then, find A and B. If the set A has 3 elements and set B has 4 elements, then find the number of elements in A x B.
(B) The cartesian product P x P has 16 elements among which are found (a,1) and (b, 2). Then find the set P.
(C) Express the function f: A-R, f(x) = x^2 - 1,
where A = {-4, 0, 1, 4} as a set of ordered pairs.

Ans. (A) Here, the first element of each ordered pair
of Ax B gives the elements of set A and the corresponding second element gives the elements of set B.

.. A= {a, b} and B = {1, 3, 2}

Given n (A) = 3 and n (B) = 4

.. The number of elements in A x B is

n (A x B) = n (A) x n (B) = 3 x 4 = 12

(B) Given, n(P x P) = 16

⇒n(P). n (P) = 16

⇒n(P) = 4 -(i)

Now, as (a, 1) ∈Px P

.. a∈P and 1∈P

Again, (b, 2) ∈P x P

.. b∈P and 2∈P

⇒a, b, 1, 2∈P

(C) Given, A = {-4, 0, 1, 4}

$f(x) = x^2-1$

$f(-4)=(-4)^2-1=16-1=15$

$f(0) = (0)^2-1=-1$

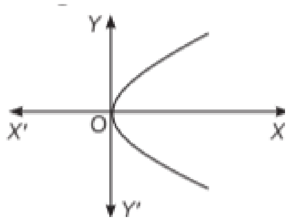
$f(1)=(1)^2-1=0$

$f(4)(4)^2-116-1=15$

Therefore, the set of ordered pairs = {(-4, 15),
(0, -1), (1, 0), (4, 15)}

3. Function as a Relation from a non-empty set A to a non-empty set B is said to be a function if every element of set A has one and only one image in set B. In other words, we can say that a function f is a relation from a non-empty set A to a non-empty set B such that the domain of fis A and no two distinct ordered pairs in f have the same first element or component. If f is a function from a set A to a set B, then we write f: A B and it is read as f is a function from A to B or f maps A to B.

(A) The given curve is a:



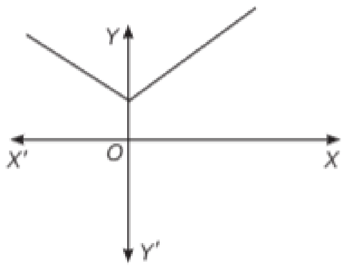
(a) function

(b) relation

(c) can't say anything

(d) data not sufficient

(B) The given curve is a:



(a) function

(b) relation

(c) can't say anything

(d) data not suffcient

(C) If f (x) = $x^2 + 2x + 3$, then among f(1), f (2) and f (3), which one gives the maximum value.

(a) f(1)

(b) f(2)

(c) f(3)

(d) f(1) = f(2) = f (3)

(D) If f (1+x) = x^2+ 1, then f(2 - h) is:

(a) h^2-2h-1

(b) h^2-2h+1

(c) h^2-2h+2

(d) h^2+2h+2

(E) Assertion (A): The cartesian product A x A has 9 elements among which are found (-1, 0) and (0, 1). The set A and the remaining elements of A x A are (-1, -1), (-1, 1), (0,-1), (0,0), (1,-1), (1, 0) and (1, 1).

Reason (R): If n(A) = p and n(B) = q, then n(A x B) = pq.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

Ans. (A) (b) relation
Explanation: If we draw a vertical line, then it will intersect the curve at two points. It shows that a given curve is a relation.
(B) (a) function
Explanation: If we draw a vertical line, then it will intersect the curve at only one point. It shows that a given curve is a function.
(C) (c) f(3)
Explanation: f(1) = 1+2+3=6,
f(2) = 4+4+3=11
and f(3)=9+6+3=18. Here, 18 is the maximum value.
(D) (c) h^2-2h+2
Explanation: We have, f (1 + x) = x^2+1) On substituting x = (1-h) in eq. (i), we get
f(1+1-h) = $(1-h)^2$+1
f(2-h)=1+h^2-2h+1
=h^2-2h+2
(E) (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
Explanation: We know that, If n(A) = p and n(B) = q, then n(A x B) = pq From the given,
n(A x A) = 9
n(A) x n(A) = 9,
n(A) = 3
The ordered pairs (-1, 0) and (0, 1) are two of the nine elements of A x A. Therefore, A x A = {(a, a): a € A}
Hence, -1, 0, 1 are the elements of A From (i) and (ii).
A = {-1, 0, 1} ..(ii)
The remaining elements of set A x A are
(-1,-1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0) and (1, 1).